II

THE PHYSICAL UNIVERSE OF DANTE[1]

IT is an inviting speculation as to what Dante Alighieri would have been had he lived as a contemporary of ourselves. Removed from the sound of the monastery bells which ordered the days of the Middle Ages, and plunged into the bustle and clangor of our own time, would Dante still have become the Divine Poet? According to some, the question must be answered in the negative, and in this strain Grandgent speaks of the Middle Ages:[2]

"It is the age of religion. To theology the best intellects turned as instinctively as in our day they are drawn to high finance. Had the great business organizer who has recently passed away lived in the thirteenth century, he would naturally have been an ecclesiastic, an eminent theologian, a pillar of the Church; and St. Thomas, were he now alive, might well be a railway magnate."

In spite of the existence of great captains and great bucaneers without the Church, and courts bursting with the opposing spirit, like that of Frederick II, hardly any one will contradict the fundamental idea just expressed: that the

[1] The author takes this occasion to thank Professors Vito Volterra and G. Vacca, and Miss Luisa Socci, of Rome, Italy, as well as his father and Professor H. W. Tyler, of Boston, for assistance in the preparation of this lecture. Professor F. Cantelli, of Palermo, Sicily, very kindly sent copies of his own papers and those of Professor Angelitti, referred to below.

Readers who find this subject interesting may pursue it further in the references given from time to time in the course of the lecture, and also in Dreyer, "Planetary Systems," Cambridge, 1906, and Orr, "Dante and the Early Astronomers," London, 1914.

[2] Grandgent, "Dante," New York, 1916, page 23.

apex of thirteenth-century life was religious. On the other hand, I can no more imagine St. Thomas Aquinas as John Pierpont Morgan than St. Dominic as Rockefeller, or St. Francis as a Guggenheim. All this may be taken without maligning our "predatory geniuses" or keeping them permanently out of heaven; but even in those realms Dante himself has made the classification for us. For the various orders of virtues and excellences are expressed by the succession of planetary and starry heavens. In the heaven of the middle planet, the sun, are found the three seekers of truth and dispensers of light, Aquinas, Dominic, and Francis; while Charlemagne is assigned to the shining cross of the warrior spirits of Mars, with Joshua and Godfrey of Bouillon; and other men of action and "executive ability" dwell in Mercury or Jupiter, some lower and some higher than the sun.

The virtues of St. Bernard are greater than those of Aquinas. And it is there that Dante is representative of the Middle Ages. It is a conscious attitude of mind with him that Faith is a greater source of knowledge than Reason, and that Science is merely the servant that arranges and accommodates phenomena to the great scheme of truth which Faith has grasped. Naturally, then, those who arrive at knowledge in the roundabout way do not radiate the same celestial light as St. Bernard.[1]

If Dante lived at the present time we might well ask if he would still consider the process of knowledge unchanged and the same order of virtues in the heavens; but hardly could we ask if there were no distinction between them. Whether to-day the poet would have written the "Divine Comedy," or a substitute for it, is a question; but whether he would have written it in his own day, if he had escaped

[1] "Paradiso," XXXI.

exile and isolation, is exactly the same question. If we think of him as too completely conscious of his own age, too thorough a spiritual representative of it, not to come to blows with it, we must think of his uncompromising will in the same relation to our own times; and it would be as reasonable to imagine him now living as a passenger of the *Buford* or behind the steel bars of Leavenworth, as within the brass cages of Wall Street. Let us not pretend that because the life of a financier is the one that provokes the greatest popular regard and envy, it is therefore the one that appeals to the keenest mind and is filled with the deepest meaning.

Before his exile Dante had demonstrated his extraordinary ability and his interest in both poetry and science; and it was merely a question of his particular fate how far they would develop. Both traits of mind were involved in practically everything he wrote. And therefore no reader can follow adequately a work like the "Divine Comedy" without some knowledge of the mechanical frame on which it is consciously built, some understanding of the physical aspect of the Medieval Universe. For here Dante himself was an expert, and he was not spending his own energy for lazy minds.

A typical instance of this activity of his own mind is in the chronology of the voyage, from the wild wood of the first canto of the "Inferno" to the infinitude of the "Paradiso." The question of the year has been difficult to determine, whether 1300 or 1301, and the question of Dante's intention in the matter, strange to say, seems to be more easy to settle from the astronomical data of the poem than from the historical references to personages and dates. Thus, to take one of the most striking and apparently precise references to dates,[1] we read in Cary's translation:

[1] "Inferno," XXI, 112–114.

> Yesterday, later by five hours than now,
> Twelve hundred threescore years and six had filled
> The circuit of their course, since here the way
> Was broken—

The earthquake referred to is caused by the entrance of the Saviour into hell.

What could be simpler than to add 34 to 1266 and obtain 1300, the date of the legendary voyage? And yet it is not simple. In the first place, the reading of the Italian text is itself doubtful, and it is just as likely that the number of years quoted is 1267 as 1266. Then, if one cares to know the day of the week and month, one must decide whether to use the year of the seasons, or the sidereal year, or the sort of year by means of which Easter is fixed. One must also remember that according to a common usage in Florence, even as late as the time of Vasari, the year ran on until Easter. After the choice is made as to a suitable sort of year, all that remain to be fixed are the dates of the birth and death of Christ, or, rather, what Dante understood they were, or intended them to be, in allegorical fashion.

One would suspect from the allegorical significance of the life of Christ that Dante would have supposed Him to die in the thirty-fifth year of His life, as counted from the Incarnation—for this is precisely what Dante writes in the "Convivio."[1] Even then one must decide upon the location of the year zero, a common conception of medieval times being to set the time of the Nativity during the year after January first of the year one. Apparently then, with respect to this reference, one may fix on any of the years 1299, 1300, 1301, and even 1302, and thus, *parti pris,* adjust the time schedules to match. The same seems to be true of the other historical references.

[1] "Convivio," IV, xxiii.

On the other hand, when one turns to astronomical references, there is much more definiteness. The morning star and the positions of the sun from time to time are given with such precision that no doubt is left in the reader's mind that Dante intended a definite sequence of days for the "Commedia." It seems possible to reconcile all the data with the year 1301, and the days from the twenty-fifth of March to the second of April, which is Easter.[1] It has been ingeniously proposed that Dante consulted an almanac which was misleading in regard to its table for Venus, and that intending the year 1300, he used the data of this planet for 1301 and the rest for 1300.[2] Although the rest of the astronomical data fits better with the year 1300 than does Venus as a morning star, nevertheless Angelitti remarks that with none of the possible days of 1300 do the references in the poem to the positions of Saturn and Mars satisfactorily accord.

The outcome of the discussion now seems conclusive. Yet whatever the outcome, there is no doubt in the central fact that there was no dissociation from Dante's mind of the world of science.

The central science of the day was astrology, and the emphasis which it placed upon astronomy tended to put the purely natural sciences in second place. Although this situation is rather reversed in the modern world, I shall return to it for the present discussion, since it is the mathematical sciences and astronomy which are most difficult for the lay

[1] Angelitti, Sulla data del viaggio Dantesco, two Memoirs read to the Accademia Pontaniana, Naples, 1897 and 1898. The positions of the planets are given in detail for the interesting dates by F. Cantelli, in two Memoirs, La Conoscenza dei Tempi nel Viaggio Dantesco and Efemeridi del Sole, della Luna, etc., republished respectively in the Pubblicazioni del R. Osservatorio di Palermo, Nos. 36 and 37.

[2] Benini, Gli enigmi di Dante, Rome, 1920, page 67, the particular reference being to the Almanacco Perpetuo di Profazio.

reader to understand from the context of the work which he is reading. Chemistry—or alchemy—is in its rudimentary stage, with wine the chemical compound of sunshine and grape-juice, and we shall not discuss it. For the reason already given we shall compress other sciences, biology, physics, and geography, to the barest indications, although the biology and the physics derive from the science of Aristotle, and are far from naïve; and the knowledge of geography has already assumed considerable proportions.

Dante briefly discusses inheritance when he treats of nobility in the Fourth Tractate of the "Convivio." It is not the stock that is noble, but the individual; otherwise, by a sort of *reductio ad absurdum,* as descended from Adam and Eve, we should all be noble or base together. The philosopher (who is Aristotle) would declare that there is only one single essence in man, and this essence cannot have different sources.[1] The diversity of individuals comes from the fact that the "complexion of the seed may be more or less good, and the disposition of the sower may be more or less good, and the disposition of the Heavens for this effect may be good, or better, or best (for this varies on account of the constellations which are always changing)."[2] Thus to explain the variety in man, the disposition of the sky takes the place of heredity. On the arrangement of the planets instead of the arrangement of chromosomes it was thought that the characteristics of the individual depended; and it was thought so for reasons identical with those used now to establish a different theory. For this pseudo-science was

[1] "Convivio," IV, xv.

[2] "Convivio," IV, xxi. Also IV, xx—"When the seed of man falls into its receptacle, namely, the matrix, it carries with it the virtue of the generating soul, and the virtue of the heaven and the virtue of the elements bound up in it, that is to say, its complexion, and it ripens and adapts the material to receive the formative virtue, which is given by the soul of the generator."

the result of experimental data obtained from observations continued over twenty-five hundred years. And now we have discarded it altogether.

I am not trying to urge the correctness of the theory, but to make it seem less fantastic by bringing it into line with the general reliance on astrology as an explanation—much as we tend to rely on mechanics. Of course there is a gap of millions of miles between the planets and the seed, but there is even yet a gap between the chromosomes and the finished product.

The community of man with nature was recognized, and therefore emphasis was placed on reason as a distinguishing attribute of humanity and a noblest part. The reasoning faculty cannot occur without the sensitive, which is common to man and beast; and the sensitive cannot occur without the vegetative, which is common to all living things. Therefore those which have the appearance of man but do not possess the reasoning faculty are not really men.

This community in living things finds further expression in a concept of continuity. There was supposed to be a continuous gradation of species. Dante also looked for a missing link, but looked upward, the downward specimens being perhaps too obvious. He assumes that there is some human being so noble and of such lofty condition that he can be hardly anything but an angel, for if not, the human species would not be continuous in both directions, which is impossible.[1] This arises from the fact that the "excellences" which proceed from the "intelligences" who move the stars come in all sorts of combinations and varying proportions, and therefore admit an infinite variety.

We must think of the manifestations of these intelligences as something akin physically to the influence of the force of

[1] "Convivio," III, vii.

gravity. The angels or intelligences by their thought (not by pushing or seeing or hearing, which senses these personifications did not necessarily possess) moved the spheres which contained the various planets, and distributed by means of them their influences through the essences of all things. On the one hand the forces of nature were derived from the thinking of the intelligences; on the other, these latter were abstractions from the former, occupying definite situations in the order of the universe. They were the personifications and guarantors of this rational order.

But this is matter more suitable for a different lecturer. The heaven that lay about the Middle Ages gave many short cuts to the systematizing of nature, and created a universal science which we cannot emulate to-day. Thus although many facts in biology, like the process of growth, and many facts in physics, like the law of the reflection of light, were known, the general idea of science was to discover "noble" properties, and use them for means of classification, thus to reconcile nature to the truths which Faith had already established. The pure forms of substances were the noblest, and the mixed were attracted by their several natures to the sources of these. Matter was attracted toward the center of the earth, its gravity being measured by this attraction. Fire, on the other hand, rose to its heaven, the Empyrean, the outermost of the heavens.

These considerations and the symmetry of the earth form the basis of arguing the question as to whether the surface of the water, which one sees sloping up toward the horizon, is or is not anywhere higher than the emergent continents. The "Quæstio de Aqua et Terra," which was brought to light in 1508 by a Friar Moncetti as a work of Dante, nearly two hundred years after a supposed debate which it reassumed, maintained the negative side of the question. The

arguments, as they are developed in detail, sound more modern than the subject suggests, so much so, indeed, that one critic cites nine anticipations of modern science, including universal gravitation, which are contained therein. Hence others conclude the book cannot possibly have been written by Dante, but was the forgery of this bombastic, self-assertive, clever, stupid, intelligent, ignorant, mathematical friar. As for myself, if these really were anticipations of modern science, I should be more inclined to give Dante the credit for the ability to make the nine generalizations than Moncetti, even if the latter is endowed by his critics with so many contradictory qualities. For these things would be as much of an anticipation by Moncetti in 1508 as by Dante in 1320. Possibly a complete determination of the authorship would be easier to make by means of philological tests, and of these I am not competent to judge. Whether or not the authorship of the essay remains unsettled, it can be safely asserted that there is nothing there which is unreasonable to expect, or cannot be found elsewhere in Dante.[1]

Before leaving this general subject, I should perhaps point out that Dante himself made some contributions to speculation about the atmosphere. He describes in the ascent of Purgatory the various strata that he goes through, passing beyond the altitude of storms, then beyond the region that contains any vapors whatever, to a space of pure immobile air, and still higher, at the terrestrial Paradise, to a region where there is a continual wind from east to west.[2] Of course the theory of the *primum mobile* which formed the basis of this speculation has been discarded in favor of modern conceptions like that of the conservation of energy;

[1] See the introductory remarks to the translation by A. C. White, Publications of the Dante Society (of Cambridge, Mass.), Report for 1902, Boston, 1904.

[2] Benini, *loc. cit.*, page 215.

nevertheless, we are here provided with an interesting illustration of how a theory which has been developed to fit certain facts will carry a number of others with it in its train.

The general science which served to unify the others, as we have already noticed several times, was astrology. We have seen how biology and physics continually lead into it. Astrology is perhaps the oldest of the sciences and extends back three thousand years and more before Christ. It flourished with continually increasing splendor through Babylon, Greece, Rome, Arabia, and Europe, almost until modern times. But a few centuries ago astronomy deserted it, and dealt it a death-blow. By the time of Kepler the order of the two sciences was inverted: astronomy was the wise mother, astrology the foolish daughter.

Astrology is supposed to be the collection of data concerning the relation of man to the heavens. The fertility of soil depends on sun as well as rain. Storms and floods do mischief and damage and seem to depend on the moon and the seasons. Thus in the early history of the human race these forces were personified into divinities of the sun, moon, and planets, and a great body of data was collected about them.

A science may progress either by becoming more simple or by adding complexity. When once a full moon in a cloudy sky brings victory in arms or a necessary rain, it is regarded by astrology as a "good sign." And if with this sign an evil event happens, then observation is made of the new element introduced into the combination, and, like any other science, the science has progressed by adding complexity. Obviously the science cannot be disproved, because the possibilities of combination are not finite. This also is not the refuge of astrology alone. Moreover, it continues to grow in prestige and fertility with the gradual discovery of certain relations of order in the heavens which become more and more certain

with the passage of time. These relations in fact have come to form the modern subjects of astronomy and meteorology. But at the time of Dante, the cosmos could be unified by a Christianized astrology, with angelic intelligences to rule the heavens of the various planets.

Before we turn finally to astronomy and the detailed motions of these heavens and their planets, let us examine Dante's possible knowledge of our earth.

The geography of the complete world was just developing. Civilization, cradled in the Mediterranean at its birth, was now seeking reacquaintance with a far-flung branch in the east and also adventure in the west. In the one direction was a country which bore strange animals and monsters, but whose human inhabitants had burst through the bonds of unreality with a demand for enlightenment and conversion; in the other, the tide of the vast unknown was fringed with lands without people, lands of ghosts and sources of terrors. We can place Dante with respect to the science of geography when we reflect that Marco Polo was his contemporary, that the Canaries were rediscovered a century after him, and that two centuries later, after a period of commercial awakening but mental stagnation, Columbus set his helm resolutely and held it fast until he had placed reality far beyond any conjectures of medieval times.

The first voyage of the Polos took place when the two brothers, Maffeo and Nicolò, the latter being the father of the famous Marco, about the middle of the thirteenth century, made an adventurous journey eastward from Constantinople. They found their return journey cut off by a war, and therefore proceeded even farther, to the famous city of Bokhara. There they met envoys from the great khan of the Tatars, who invited them to again continue their journey a full year further, in order to let the eyes of the great khan

at Peking fall upon members of the Latin race. Kublai Khan gave them letters to the Pope, requesting a delegation of learned men to expound to him the Christian law and culture.[1] The two brothers ended their first voyage at Acre in the year 1272.

On the second voyage, which began about two years later, young Marco went along, and with his skill in diplomacy and facility in learning languages became a favorite of the great khan. He went on numerous expeditions through China, observing everything; he visited Burmah and Japan, became a governor of a Chinese province, and discovered Prester John. The Polos altogether made themselves so indispensable that it was only an opportune embassy to Persia that finally persuaded Kublai Khan to let them act as guides and return to Venice, after an absence of about twenty years.

Of special interest is the early mention of petroleum in Armenia, and a rapid passage from Noah's Ark to Standard Oil. "Further I say that in Great Armenia rests the Arc of Noah on top of a high mountain in the southeastern part, in a region called Mosul, where the inhabitants are Christian Nestorians and Jacobins, of whom we shall speak later. In the West the province is contiguous with Georgia, and near this boundary is found a great spring, from which oil rises in such abundance that a hundred ships could be loaded at one time with it. It is not good to eat, but to burn, it is good for the itch and other things. Men come for it from afar, and through all the region no other oil is burned."[2] In this book one finds the dreams of Coleridge,—Xanadu and Kublai Khan and the Great War.

All this was known well enough in Venetia to give Marco

[1] Six or a hundred, according to the manuscript cited.

[2] Polo, "Il Milione, a cura di Dante Olivieri," Bari, 1912, page 17.

Polo the nickname of Milione,[1] from the golden stories that he told, and Dante, spending the closing years of his life in the neighborhood, could not have been ignorant. In fact Europe and Asia, with the northern part of Africa, comprise the emerging continent which is described in the "Quæstio de Aqua et Terra," extending from Spain to the river Ganges and from the equator to the arctic circle, roughly in the shape of a half-moon.

A striking error in Dante's notion of the civilized world is that of making the length of the Mediterranean extend for ninety degrees of longitude—perhaps an intentional remodeling of geography to fit allegorical interpretations. In the geography of the Middle Ages, distances were apt to be somewhat indefinite, and therefore the maps which are left to us as relics seem at a first glance singularly immature and inaccurate. And yet we must remember that a map is designed to show order as well as magnitude, and the medieval maps might show order very well, and the actual order and even distances might be known very well, without any suggestion of the pink, green, and purple areas that are so familiar to us. In fact, in modern maps of large areas, distances are only represented by conventions, described in terms of various projections; and this point of view could not be developed until after the application of geometry to perspective. If we remember the history of this application and its slow development in the painting of the Renaissance, we need not wonder further at its absence from early maps.

The geometrical properties of the earth's surface, regarded as the surface of a sphere, were, however, fully conceived. Witness the geodesy of the Third Tractate of the "Convivio,"[2] where the relative positions of poles, equator,

[1] See the account given in H. G. Wells's recent History.

[2] "Convivio," III, v.

and ecliptic are discussed, and the relation of day to night. Here incidentally the radius of the earth is given as thirty-two hundred and fifty miles.

When we turn to the west, there does not seem to be a general knowledge of explorations in that direction. The question is not whether the Norse had discovered America, but whether the Italians were sufficiently aware of it to know that there was an actual boundary to the sea which stretched beyond the gates of Gibraltar. It is true that the Norsemen and Icelanders had made long journeys and colonized Greenland sufficiently to make the collection of church tithes worth while; and we have letters dated from the dawn of the thirteenth century to the end of the fifteenth, written by various Popes for this purpose.[1] And then, precisely at the time when the Italian ventures were about to commence, the Norse ventures declined, and through the commercial policy of their overlords even Greenland was cut off.

In several of these papal letters written before and during the life of Dante, explicit mention is made of numerous settlements, presumably in Greenland, the name of which is also explicitly mentioned. In one, of date well after Dante's time (1448), reference is made to an invasion of Greenland thirty years before that date, when barbarians "from the pagan shores beyond" had destroyed practically all the settlements. From such letters it might perhaps be admitted that the existence of a large land in the west, corresponding to America, was not beyond the surmise of well informed people of southern Europe, even in the thirteenth century.

Some commentators have insisted that back of the passage which in the XXVIth canto of the "Inferno" describes the well known voyage of Ulysses, there was in Dante's mind some such conception. Here is Johnson's translation:

[1] See the publications of the Norroena Society, New York, 1906.

Myself and my companions had grown old
And slow, when we had reached that narrow strait
Where Hercules had set his boundaries,
In order that man put not out beyond;
Seville I left beyond upon the right,
With Ceuta passed already on the left.
"O brothers," said I, "who are come at last,
A hundred thousand perils undergone,
Into the west, to that which still remains
Of this, your senses' vigil, now so brief,
Do not deny experience, with the sun
In front of you, of the unpeopled world.
Consider of what origin ye are;
Ye were not made to live as do the brutes,
But to seek virtue and to learn the truth."
With these few words addressing them, I made
So eager my companions for the voyage,
That I could scarcely then have held them back;
And when our stern to the morning had been turned,
The oars became our wings for that mad flight.
As we went, ever gaining on the left,
The night already looked on all the stars
About the other pole, with ours so low
That it rose not above the ocean floor.
Five times rekindled and as many quenched
Had been the light beneath the moon, since we
Had entered on the passage of the deep,
When there appeared to us a mountain, dark
Because of distance, and it seemed to me
Of such a height as I had never seen.
We felt great joy, but soon it turned to grief
Because a whirlwind rose from that new land
And struck our ship upon the forward part.
Three times it made her whirl around with all
The waters, and the fourth, lifted the stern
And downward sent the bow, as pleased Another,
Until the sea again closed over us.

From the tone of the quotation it is evident that Dante praises the adventurous search for knowledge, and does not regard Ulysses as tearing presumptuously at curtains which should not be parted. This fiery passage with its tragic climax may well have formed part of the inspiration of Columbus, two centuries later.[1] And yet the weight of opinion seems to confirm the judgment that it is not a mysterious new land upon which Ulysses is wrecked, but the legendary Mountain of Purgatory. In fact, the wooden horse of Troy has already been made the bearer of this hero's destiny, and he can never reach the slope that leads to salvation.

In Dante's geography, Purgatory is a mountain exactly opposite Jerusalem in latitude and longitude, reaching into the air perhaps as much as one hundred and forty-seven miles.[2] This height might almost be called a passage from geography to astronomy, and we shall make the transition at this point, because we do not wish to consider Hell and Purgatory as parts of the physical universe. In fact, given the long chain of precursors of Dante in making the geography of the regions of lost and saved, given the conscious mixture of classical and Christian legends in Dante's plan, and its rational value as Dante's theoretical analysis of sin and salvation, it is hardly reasonable to suppose that Dante himself, with his clear ideas of time and space, wanted to make his supernatural geography natural and append it all to the Creed!

The astronomy of the time was practically the astronomy of Ptolemy (about A.D. 137), although not nearly so much could be spoken of as current in the cultured circles of the

[1] G. Finali, "Cristoforo Colombo e il viaggio di Ulisse nel poema di Dante," Città di Castello, 1895.

[2] Benini, *loc. cit.*, Chapter VII.

Middle Ages—and this although Arabic culture already was beginning a reflourishing of knowledge in Spain. Perhaps it might be more correctly described as the astronomy of Hipparchus. Interpreted in the light of modern knowledge, it was merely an approximate representation of apparent angular motions of stars and planets as seen from the earth, by means of a system of uniformly moving epicycles and eccentrics. It was thus an attempt to build up the variable apparent motions of the heavens out of a system of uniform circular motions about centers themselves also moving uniformly.

The simplest example of this theory is perhaps the case of the sun as analyzed by Hipparchus, where the result was so accurate that it was not until the invention of the telescope, in the time of Galileo, that there was seen the necessity of the slightest change. Before considering in detail this theory, which makes the apparent position of the sun

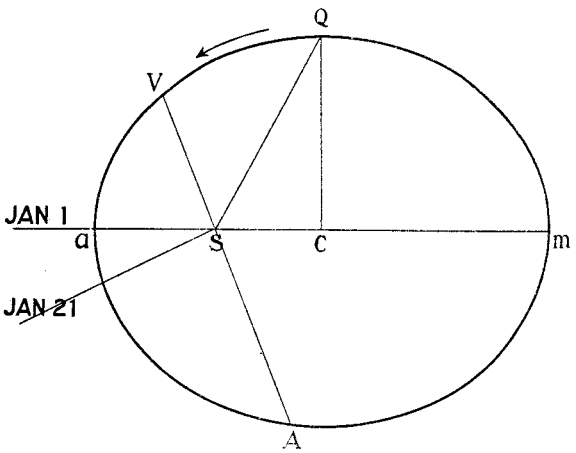

Figure 1. S is the sun; V, position of earth at autumnal equinox; A, position of earth at vernal equinox; CQ, semi-minor axis; Cm=SQ, semi-major axis; $SC/SQ=e=1/60$, $SC^2/SQ^2=1/3600$, but $SC^2=SQ^2-CQ^2$, therefore $CQ^2/SQ^2=1-1/3600$ and CQ/SQ $1-1/7200$.

accurate throughout its course to less than a minute of arc, let us consider the situation as we now know it. In the first place, the orbit of the earth about the sun is an ellipse, very much exaggerated in Figure 1, the sun being at one focus. The major axis of the orbit determines the two points a and m, called perihelion and aphelion respectively, the line joining them being called the line of apsides. The ellipticity of the orbit is measured by a number called the eccentricity (e=SC/SQ), in the case of the earth about 1/60. This means that in the case of the earth the short and long radii of the ellipse (the minor and major semi-axes) differ by only 1 part in 7200.

For the earth to travel from perihelion to perihelion takes 365 days, 6 hours, 13 minutes, and 49 seconds, a period of time which is called the anomalistic year, distinguishing it on the one hand from the sidereal year of 365 days, 6 hours, 9 minutes, and 9 seconds, and on the other from the tropical year of 365 days, 5 hours, 48 minutes, and 46.15 seconds. We do not ordinarily measure time by the anomalistic year, and the reason lies with the peculiarities of this orbit. The line of apsides is not fixed in the sky like the fixed stars, but moves slowly ahead in the direction of the earth's motion in its orbit, by some 11.5 seconds of angle per year, an effect due mostly to the attraction of the larger planets. In other words, if one year when the earth is at perihelion (which nowadays happens on January 1) we sight a star in the direction Sa, and the next year notice when the same star is in the line joining the sun and the earth, the earth will not again be at *a,* for the point *a* has moved on again by 11.5 seconds of arc. We catch up to the star again in a period shorter by 4 minutes and 40 seconds than the anomalistic year. This is called the sidereal year.

As is seen by the calculation previously given, the displace-

ment of the sun from the center of the orbit (1 part in 60) is more pronounced than the variation of the orbit from a circle, measured by the difference in major and minor axes (1 part in 7200); and therefore an eccentric circle gives a good substitute for an ellipse where the sun is a sixtieth part of the radius away from the center. (See Figure 2.) This in fact is the basis of the eccentric theory of the planets, except that a further inversion of the modern theory is present;

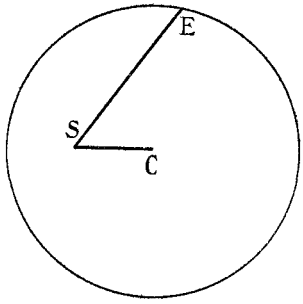

Figure 2. E is the earth and S the sun; CE=r, SC=er, e=1/60.

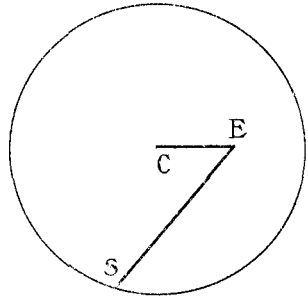

Figure 3. E is the earth and S the sun; CS=r, CE=er, e=1/60.

namely, that the earth is taken as the center of motion and regarded as at rest. We have then a picture of the apparent motion of the sun as portrayed in Figure 3. The sun is assumed to move at uniform speed around the circumference, the earth being, however, not at the center but displaced from the center by a distance 1/60 of r.

Another geometric interpretation which yields exactly the same apparent orbit for the sun is obtained by an epicycle. This is a circle whose center moves on another circle, called the deferent. We can replace Figure 3 by Figure 4, where E is taken as the center of the circle of radius r. On it there travels with uniform velocity the center of another circle, of

radius er; the sun this time lies on the circumference of this second circle. If the line PS keeps parallel to the fixed line EO, and P moves on the deferent with a constant velocity, then (as is seen by shifting the deferent by the amount EO) S lies on the eccentric of the previous figure, and moves along it with constant speed.

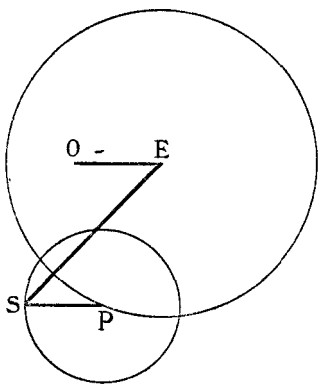

Figure 4. E is the earth and S the sun, EP=r, SP=er, e=1/60.

The time of a complete circuit, according to the theory of Hipparchus, was taken as a tropical year. But this again is a third kind of year, which we shall take occasion later to explain. Suffice it to say that this arrangement does not take account of the yearly advance of the line of apsides, but does provide for a second phenomenon, called the *precession of the equinoxes* (an allowance of about twenty minutes of time in the year), which Hipparchus was the first to discover.

It will be seen that this general system and this particular phenomenon were well enough known to Dante to enable him to proceed accurately on his voyage through the heavens.

In order to discuss the precession of the equinoxes, let us

imagine a perspective view of the earth's orbit, regarding, as at the present day, the sun as fixed; the axis of the earth through the north pole is not perpendicular to the plane of the orbit (called the ecliptic), but inclined at an angle of about twenty-three and a half degrees, and as the earth moves round the orbit this axis remains very nearly parallel to itself. The equatorial plane of the earth and the plane of the ecliptic make this same angle with each other. The line of their intersection, on account of the constancy of the planes, likewise remains very nearly unchangeable in direction, and determines a direction through the sun (see Figure 5) which can therefore be used as a line of reference in the orbit. It is called the line of equinoxes, the back end of which (the point A) can be seen at any time during one of the late spring starry nights, since it ends in the constellation Virgo.

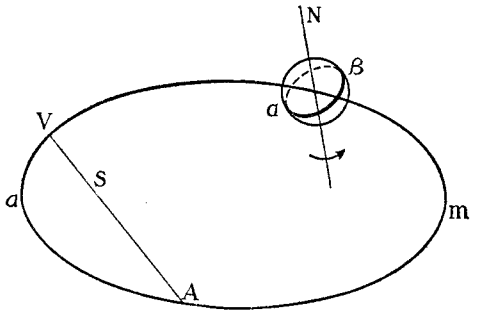

Figure 5. N is pole of the earth; $\alpha\beta$, the equatorial plane of earth; am, plane of ecliptic; VA is parallel to line of intersection of planes am and $\alpha\beta$.

You will notice from the diagram (Figure 5) that if the earth is to the right of this line, the sun, as the earth rotates around its axis, will seem to pass across the sky above the earth's equator, and it will be the summer half of the year in Texas. The part containing *a* is the winter part, although

then the earth is nearer the sun. When the earth is at V and the sun seems in the direction SA, it is September 21; and when the earth is at A and the sun is in the direction SV, it is March 21. The point V is therefore called the vernal equinox, and the time necessary for the sun to leave the direction V (as seen by us) and get back to it again is the usual year (called the tropical year), and the measure of the seasons.

We have mentioned the fact, however, that the tropical year is not the same as the sidereal year. The direction SV itself travels round with reference to the stars, or, more particularly, with reference to the constellations of the zodiac, at the rate roughly of one degree a century. It is this phenomenon which is called the precession of the equinoxes. Hipparchus discovered it by comparing his observations of the vernal equinox with those of 1200 years before him.

In terms of our modern mechanics this motion is not difficult to explain. For the earth is a spheroid, and not a sphere, and therefore the effect of the attraction of the sun is not the same as if the earth were concentrated at a central point. In fact if the form of the earth is still more exaggerated into a sort of equatorial ring, it is seen that what we have is a huge gyroscope whirling rapidly (that is to say, one revolution in twenty-four hours!) about the polar axis (inclined from the perpendicular to the orbital plane by twenty-three and a half degrees), and the sun pulls more strongly on the nearer portion of the ring. But when you pull down, say, on one side of a rapidly whirling gyroscope, the axis immediately tends to move sideways, at right angles to the direction of your pull.

This, at first sight, strange phenomenon, which is merely a consequence of the centripetal forces, is the analogue of the precession of the equinoxes. The axis of the earth tends to move slowly sideways, and instead of pointing rigidly in

a fixed direction in the sky, forms by its motion the surface of a cone whose half-angle at the vertex remains nearly the constant value of twenty-three and a half degrees. This motion of the axis of the earth produces a corresponding rolling of the equatorial plane and rotation of the line of the equinoxes. This precession of the equinoxes is backward, against the earth's motion in its orbit, and will carry the vernal equinox through a complete circuit of the zodiac in 36,000 years.

From this phenomenon the zodiac has come to have two significations. The *signs* of the zodiac refer to the distances of the sun at various periods of the year from the vernal equinox, and thus mark the twelve months of the tropical year. The *constellations* of the zodiac are understood to be the actual divisions of the sky corresponding to the named constellations. The signs of the zodiac were originally named from the constellations, at the birth of astronomy; but owing to the precession of the equinoxes themselves, the names no longer agree, but are displaced from each other some twenty-five degrees or more. To say that the sun is in the Ram, as a sign of the zodiac, is not the same as to say that it is in the Ram, as a constellation of the zodiac. Dante was quite conscious of this difference.[1]

We are now in a position to build up the physical universe of the Middle Ages, more or less even in the words of Dante himself.[2]

The earth occupies a central position at rest, and the orbits of the moon, planets, sun, and stars are fixed on concentric spheres or heavens of various radii. Of these Aristotle thought there were only eight, in the order moon, sun,

[1] Angeletti, Sugli accenni Danteschi ai segni, alle Costellazioni ed al moto del cielo stellato, Rivista di Astronomia e Scienze affini, Anno VI, three papers.

[2] "Convivio," II, iii, iv.

the several planets and the fixed stars, but Ptolemy afterward saw that "the eighth sphere was swayed by several movements," as he saw that its circle deviated from the true circle which turns everything from east to west (the apparent daily rotation). Constrained by the principles of philosophy, which demand the simplest *primum mobile,* he therefore assumed that a ninth heaven lay beyond that of the stars, and that this caused the daily revolution of the sky from east to west.

The medieval order of the spheres is this: (1) Moon, (2) Mercury, (3) Venus, (4) Sun, (5) Mars, (6) Jupiter, (7) Saturn, (8) Stars, (9) Crystalline or Primum Mobile, (10) the Empyrean Heaven, "which Christians add to the other nine, as the abode of that highest Godhead who alone completely beholds himself." This last is considered as immovable, a completely satisfied intelligence; and the "desire" of unification with it is that which in the intelligences which preside over the ninth sphere give that sphere a swift movement, and this is then distributed in various ways down through the various other intelligences and their corresponding spheres, a sort of conservation of energy.

"Now we should know (Figure 6) that each heaven below the Crystalline has two poles fixed as regards itself (as a matter of fact for every sphere nearly coinciding with the pole of the ecliptic), the poles of the ninth heaven being absolutely fixed and stationary." The movement of the eighth sphere becomes now merely one of very slow rotation (opposite to that of the *primum mobile*) about the pole P, such as to make one revolution every 36,000 years. This movement is merely the correction due to the precession of the equinoxes, allowing for the fact that the north pole of the earth does not point continually to the same constellation in the heavens, but moves relatively among the stars.

Here we are merely making the stars move relatively to it.

In each of the other spheres there is a plane midway between its two poles, and in this plane the orbit of the corre-

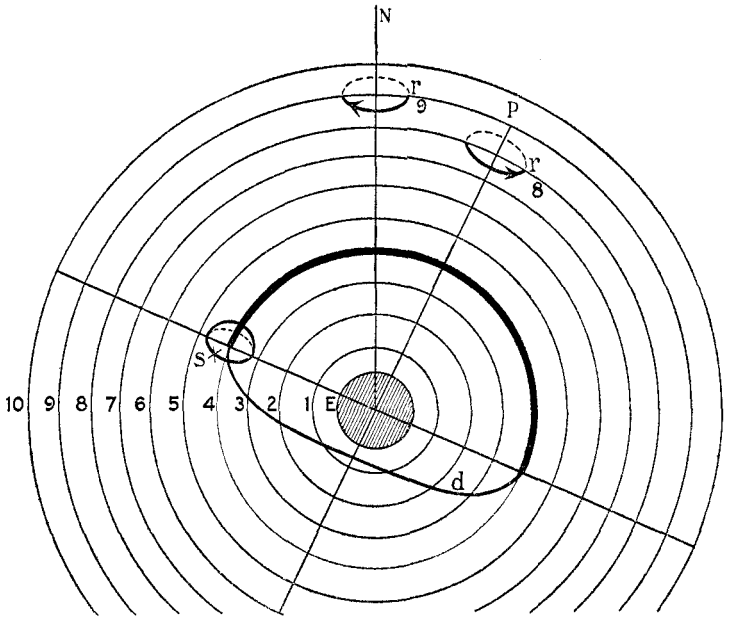

Figure 6. E, Earth; 1, Moon; 2, Mercury; 3, Venus; 4, Sun; 5, Mars; 6, Jupiter; 7, Saturn; 8, Fixed Stars; 9, Crystalline; 10, Empyrean; S, Sun on its epicycle; d, deferent of Sun, equator of fourth sphere; r_9 indicates revolution of ninth sphere, once per sidereal day of $23\frac{14}{15}$ hours; r_8 indicates revolution of eighth sphere, one degree per century, about axis which itself partakes of motion of ninth sphere; P, Pole of Ecliptic, attached to ninth sphere; N, North Pole of Earth, fixed absolutely.

sponding planet lies. The circle of the sphere in this plane is precisely the circle which in the particular case of the sun we have previously spoken of as the deferent.

On this circle (the deferent) there is another small sphere whose center moves round it, and the circle of this small sphere is what, as Dante says, the astrologers call the epi-

cycle. These epicycles are heavens by themselves, with new intelligences, so that there are really more than ten heavens, but "the epicycle is more nearly of one nature with the heaven in which its center lies than with the others, and is named after the same star."

We thus see how, by a system of deferent and epicyclic spheres or circles inside the *primum mobile* and that of the stars, the cosmology of the Middle Ages was built up, and we have studied the case of the sun in detail. The epicycle in that case provided a means of making the observed difference in the length of the different seasons. We might even have gone farther and provided for the rotation of the line of apsides (Figure 1) if we had made the radius PS of the epicycle advance by eleven seconds of arc during the year.

But with the planets the case is somewhat different. There the epicycles take care merely of the simplest motion, direct and retrograde along the zodiac, and even in Ptolemy's time this simple theory had become inadequate. Hipparchus had in fact already combined two ideas and put an epicycle on an eccentric in order to get the motion of the moon, and Ptolemy found it necessary to make the same combination even to get approximate accuracy for the motion of the planets. There is, however, another point where the simple theory of epicycles or eccentrics breaks down, and that is in the relation between velocity at different points of the orbit and distances to those points.

It is one of Kepler's well-known laws that a planet moving round the sun sweeps out equal areas in equal times (see Figure 1). Hence when the planet and sun are near, the relative velocity must be larger, and when they are farther away, smaller. But this is obviously not given to us by our hypothesis of uniform velocity around the eccentric. In the case of the sun and earth the difference would not be striking.

But for some of the planets, and especially for the moon, the radii have to be of extremely faulty proportion to get the proper velocities. And this led to an extremely important consequence.

The astronomers had developed a mathematical system which became more and more capable of extreme accuracy; and yet it did not correspond to any known mechanical or geometrical interpretation. It was a little like the Maxwell theory of electricity from the years 1900 to 1910. The equations themselves, without a mechanical interpretation, were taken as a description of the phenomena, and to a certain extent they still are. Medieval astronomers had arrived in their theoretical astronomy at this same rather advanced and abstract view.

It is time perhaps for Science to grow beyond the need of a mechanical interpretation. There is no reason why that should be the only or the most desirable formulation of the implications between phenomena—especially since the phenomena have passed to a stage of minuteness where the mechanical aspect has no direct significance. Whenever there is one mechanical explanation, the transformation theory of dynamics tells us that there is more than one, and of these the simplest, as Einstein has shown us, is the most complicated. On the other hand, when we try to classify the phenomena that admit of mechanical explanation, and Professor G. D. Birkhoff tells us that any system of ordinary differential equations is nothing but a set of dynamical equations, and vice versa, it becomes evident that the future of science may soar farther from our own restricted mechanical point of view than ours has risen above the quaint interpretations of the Middle Ages.

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